A parallel procedure for the exhaustive numerical solution of the polynomial equations

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Abstract. For the exhaustive numerical determination of the real roots of an algebraic equation with real coefficients the Sturm's sequence method can be used. The complex roots of such an equation can be derived by means of the Bairstrow method. Both methods are of a serial type. In this paper a parallel strategy for simultaneous computation of the real and complex roots of algebraic equations with real coefficients, based on a combination of iterative and randomized methods, is proposed.

1. Introduction

Consider the normalized algebraic polynomial equation, of degree n (with $p[0] = 1$), with real coefficients and without zero-roots (with $p[n] \neq 0$)

$$pol[n](x) \; = \; \sum_{i=0}^{n} p[i] * x^{n-i} = 0$$

For the determination of the complex roots we take into account the fact that in the complex plane their afixes are situated between the circles of radius $r = rinf$ and $R = rsup$ (fig. 1). Thus, if ζ is a complex root of $pol[n]$ and $a = re\zeta$ and $b = im\zeta$ are the real and the imaginary part of ζ respectively, then we have $a, b \in (-R, R)$ with $R^2 > a^2 + b^2 > r^2$. Since the coefficients of the equation are real numbers, the nonreal roots are complex conjugated. Therefor it is sufficient to determine the roots with the afixes situated in the upper plane, delimited by the x-axis.

Using the polar coordinates

$\rho \; = ro =$ polar radius

$\theta = theta =$ polar argument

we have

$a = \rho \cos\theta, \;\; b = \rho \sin\theta,$ with $\theta \; \in [0, \pi]$ and $\rho \in (r, R)$.

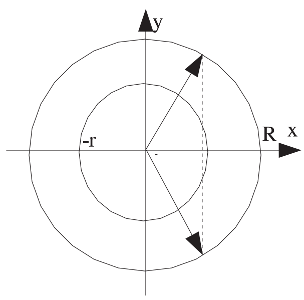

Figure 1. The relationship between coordinates.

In the search for the complex roots we use p processors in parallel: $P[1], P[2], ..., P[p]$. The interval $[0, \pi]$ is splitted in p subintervals of length π/p. Thus, the processor $P[k]$ searches the solution for

$\theta[k] \in [(k-1)\pi/p, \;\; k\pi/p], k = 1...p$

If one complex root is determined and we denote it by (a, b), then we can reduce the degree of the equation by 2, by the dividing $pol[n](x)$ by $x^2 - 2re[k]x + re[k] * re[k] + im[k] * im[k]$. The remainder of the division has the form: $r0[k]x + r1[k]$.

For this division we use the processor DIV. At the input we have the dividend and the divisor, as vectors of coefficients, while at the output we have the quotient and the remainder (which has to be equal to zero).

2. Processor for the random generation of the polar radius

The processor GDP will randomly generate the values of $\rho = ro$, using the formula: $\rho = r + (R - r) * rnd$, where rnd is a pseudorandom number generator.

Figure 2. GDP processor.

3. Processor for the random generation of the polar argument

The processor GAP randomly generates the values $\theta[k]$ using the formula $\theta[k] = (k-1)\pi/p + (\pi/p) * rnd = theta[k]$.

Figure 3. GAP processor

4. Processor for the determination of the real and imaginary parts

The proccesor PRI uses the transformations:

$$re = ro\cos(theta),\ im = ro\sin(theta),$$

$$with\ theta \in [0, \pi] and\ ro \in (rinf, rsup).$$

FIGURE 4. PRI processor

5. Parallel procedure for the derivation of the non-real roots

```
procedure com_par(n,pol,eps,p);
select eps,p;

begin
    a:=rinf(n,pol);b:=rsup(n,pol)
    poz:=vars(a,b); neg:=vars(-b,-a);
    com:=n-poz-neg;
    m:=0;

    while (m <> com/2)
        for k=1 to p do in parallel (asynchronous)
            ro[k]:=rinf+(rsup-rinf)*rnd;
            teta[k]:=(k-1)*pi/p+rnd*pi/p;
            re[k]:=ro[k]*cos(teta[k]);
            im[k]:=ro[k]*sin(teta[k]);
            call div(pol,imp[k],n,2);
            if(r0[k]=0 or r0[k]<=eps) and
                 (r1[k]=0 or r1[k]<=eps)
            then m:=m+1;
    endif;
        endfor;
    endwhile;
end.
```

References

[1] I. Dzitac, *Parallel computing,* Ed. Univ. Oradea, 2001 (in Romanian)

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